# Counting Rectangles

Extra Scene from "The Math Olympian"
Sophia Matthew, Quest University Canada

"Do you remember the rectangle problem?" asked Mr. Collins.

"You mean the one where we counted the number of rectangles in a 3×3 grid?" I asked, pulling off my red Team Canada mittens and setting them on the heavy wooden table.

I glanced around Le Bistro, one of my favourite spots in Cape Breton, especially on a snowy December day like today, where I could be inside with Mr. Collins and learn math. I heard the steamer's pitch climb higher and higher as Joe prepared my drink. Today, being almost Christmas, I'd ordered a peppermint hot chocolate.

I sipped my drink and let it warm my hands. "So, the rectangle problem?" I asked.

"Yes," he said. "Let's go back to it."

"Okay," I said. "But Mr. Collins, haven't we already done this?"

"Not by a long shot, Bethany," he said, with a smile. "Not by a long shot." Mr. Collins' brown eyes met mine. "I want you to understand that one problem can have many layers. With the problem of counting rectangles, we've just scratched the surface."

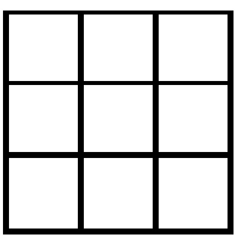
He grabbed an index card and a pencil from the tatty leather briefcase that seemed to be an unfortunate staple of his wardrobe and slid them across the table.

"Bethany, I want you to draw a 3×3 grid."

"Like before?" I asked.

"Precisely," said Mr. Collins.

Of course, I knew how to draw a 3×3 grid and besides, we'd already solved this problem.



"Okay, Bethany, remember what we did before?"

"Yeah," I said, "I thought I'd found all the cases just by counting them, but you showed me that I'd actually forgotten a case. I think it was the 3×2's."

"And what did you learn from that?"

"I learned that just *thinking* I know isn't good enough, I have to *know* I know. I can't guess, even if I think a problem is obvious."

"Excellent."

Mr. Collins surveyed the occupants of Le Bistro: two couples sipping coffee, four students sitting around a table with textbooks open and notes scattered, and a mother trying to get her crying baby to eat some banana bread. Mr. Collins turned back to me, "Now I'm going to ask you a totally different question. If you could travel anywhere in the world, where would you go?"

I paused. "England."

Mom and I didn't have enough money to travel, so I'd never actually been on a vacation and I wasn't sure where I'd want to go. Travelling wasn't something I thought much about.

"England," repeated Mr. Collins, "now for this math problem, you have been given five plane tickets to England. That means you have four tickets in addition to your own. You decide to bring four friends. Do you understand the problem?"

"Yes."

"Great. Pretend you have eight friends, four girls and four boys. Your female friends' names are: Anna, Babette, Clarissa and Daphne. Your male friends' names are: Edmund, Ferdinand, Glen and Hamlet. You want to give your plane tickets away to two of your female friends and two of your male friends."

"Sure."

"But let's say you want to count the number of different ways you can give out the four tickets. For instance, you could give your tickets to Anna and Clarissa, two of your female friends, as well as Edmund and Hamlet, two of your male friends."

I nodded.

Mr. Collins continued, "now, let's come back to our grid. How could we use it to help?"

I thought about the information I'd been given: four female friends, four male friends and four tickets to England. I looked at the grid. It was a 3×3 grid. The number three wasn't anywhere in the problem.

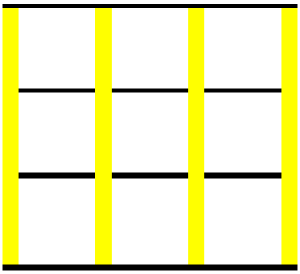
I glanced out at the snow. I wasn't sure how the grid helped. The baby was still crying as the mother talked to him in some foreign language; I wasn't sure which one.

"Any ideas, Bethany?" asked Mr. Collins, bringing me back to reality.

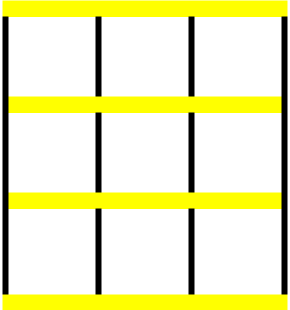
"Mr. Collins, I'm not sure what to do. I understand the grid and I understand the problem with the tickets but not how they relate."

"Okay, Bethany, I'm going to give you a hint." Mr. Collins took the piece of paper from me. He drew two new 3×3 grids, and then reached back into his briefcase. He grabbed a yellow highlighter.

On the first new grid, he highlighted all the vertical lines.



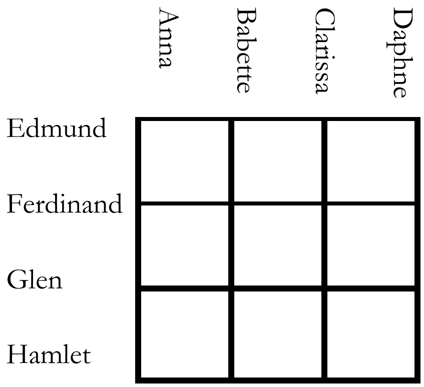
On the second new grid, he highlighted all the horizontal lines.



Mr. Collins gave the paper back to me. I looked at all the grids, the unmarked original and the two new ones with highlighted lines. I sipped my peppermint hot chocolate and drummed my fingers on the table. I still didn't know how the grids were connected to the problem of my friends and a trip to England.

Then it hit me, all of a sudden. A 3×3 grid was made out of *four* horizontal and *four* vertical lines. And I had *four* female friends and *four* male friends. It was perfect!

I looked at Mr. Collins and smiled but didn't say anything. I picked up a pen and wrote the names of my "friends" on the index card.




"Fantastic," said Mr. Collins. "Now, any guesses what we're going to do next?"

"Figure out which friends get the tickets to England using the grid."

"Precisely," said Mr. Collins. "But first I want you to tell me this: how many ways are there to give two tickets to your four female friends?" He passed me an index card.

I wrote down all the possibilities I could think of:

ANNA, BABETTE
ANNA, DAPHNE
BABETTE, DAPHNE
ANNA, CLARISSA
BABETTE, CLARISSA
CLARISSA, DAPHNE

I figured the order within the pairs didn't matter, since giving two tickets to Anna and Clarissa would be the same as giving them to Clarissa and Anna.

"Perfect," said Mr. Collins. "We can easily see that there are six pairs, and that you've considered every case."

"Okay."

"You'd agree with me that if there are six ways of giving two tickets to your four female friends, there would also be six ways of giving two tickets to your four male friends?"

"Yes, I would," I said. This made sense because there was nothing special about the example of female friends. I reasoned that picking two things out of four things, no matter what, could be done in six ways.

"We have six different ways to give two tickets to our female friends and six different ways with our male friends. So, here's my question: how many different ways can you give the four tickets to your friends?"

I remembered a question Mr. Collins had given me a couple of weeks ago. It had involved ice cream and sprinkles. We had reasoned that if you had $x$ types of ice cream and $y$ types of sprinkles, you could make $xy$ different combinations (if you used one type of ice cream and one type of sprinkles). This same idea could be used to calculate the number of combinations of anything. All you had to do was multiply the numbers of different items in the first category by the number of different items in the second category, or $x$ times $y$, making $xy$.

For this problem, I knew if I multiplied the number of possible ways to give tickets to my female friends by the number of ways to give tickets to my male friends, I would get the answer. So, there were 6 times 6 different ways to give the tickets to my friends.

"36," I said.

"Excellent," said Mr. Collins. "Now remember, when we originally did this problem, how many different rectangles could we make?"

"Wasn't it also 36?" I asked.

"Do you think that's a coincidence, Bethany?"

"No," I said. I'd learned through my sessions with Mr. Collins that few things in math were coincidences, even if they appeared to be.

Mr. Collins smiled. "Well, let's look back at our grid. Why is the answer to both the original problem of counting rectangles as well as this new problem with plane tickets and friends equal to 36? What's going on?"

I looked at the grid again. "Because each friend corresponds to a line?"

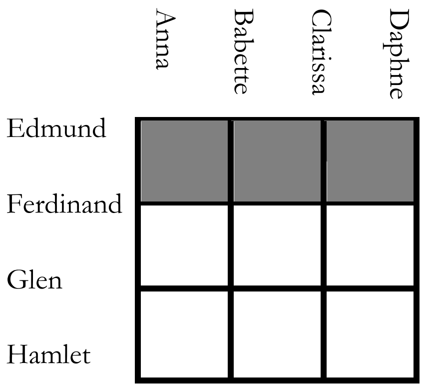
"I think you're on the right track, Bethany. Go on."

“Well, if I picked Anna and Daphne for my female friends and Edmund and Ferdinand for my male friends, I’d have selected four friends as well as four lines on the grid. Those two female friends correspond to two vertical lines on the grid and those two male friends correspond to two horizontal lines. And I could make a rectangle out of my friends but also out of the lines on the grid.”

“Can you give me an example?” asked Mr. Collins.

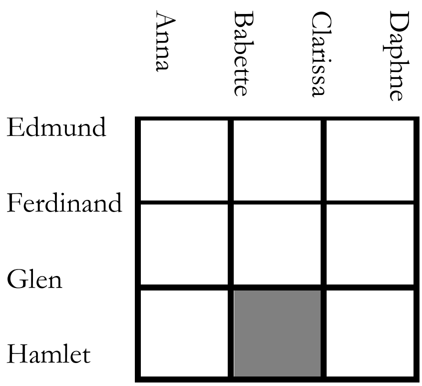
“Okay,” I said, “I could draw a rectangle formed by the lines which represent Anna, Daphne, Edmund and Ferdinand, like this.”

I shaded in a rectangle the names of my “friends” on the index card.




“Fantastic,” said Mr. Collins. “What about if you picked Babette, Clarissa, Glen and Hamlet?”

I traced a square at the bottom of the index card with my finger.



"Why does this work?" asked Mr. Collins, "Why can you go from lines to friends?"

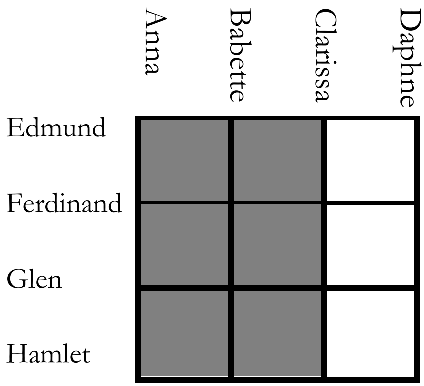
I stared at the diagram, unsure of exactly how to say it.

"Because it's sort of the same?" I ventured.

"Go on," said Mr. Collins.

"Well," I said, "This is still our 3×3 grid, but I labeled it with my friends' names. And each friend is also a line."

"Okay, let's try one more example," said Mr. Collins, "but this time I'm going to give you a rectangle and you have to tell me which friends are going to England with you. Not only can you go from lines to friends, you can go from friends to lines." He drew a new 3×3 grid, added my friends' names and drew a new rectangle on it.




It took me only a second to reply. "I'd give my plane tickets to Anna, Clarissa, Edmund and Hamlet."

I liked that it was possible to go from lines to friends and back again.

"Good work, Bethany, I think you get the point," said Mr. Collins. "By picking two female and two male friends, you are also picking two vertical and two horizontal line segments. In our original problem, a couple of weeks ago, we drew a rectangle comprised of two vertical lines and two horizontal lines. Both are exactly the same problem, but they are presented in two different ways. We've just changed the *context*. Knowing there are multiple ways to view a problem can help change your perspective. This allows you to solve a problem in different ways and with more confidence."

"That's pretty cool," I said.

“Yes, it is and I’d like you to write up a formal proof for next week about why it works to switch from friends to lines.”

I took sip of my now-lukewarm peppermint hot chocolate.

“Now, I want to make sure you understand some terminology,” Mr. Collins said. “When you have four friends and you pick two, we can call it *four choose two*. Does that make sense?”

“Yes,” I said.

If you had four things and picked two, it seemed logical to call it ‘four choose two’.

I asked, “If we had instead five things and picked two that would be *five choose two*, right?”

“Yes, it would. Now, let me show you how to write it,” said Mr. Collins. He scribbled a pair of brackets on the piece of paper. Inside he wrote two numbers, one on top of the other.

$$\binom{5}{2}$$

“That’s how you write *five choose two*,” said Mr. Collins. “The number on top is the number of possible things to pick from, or five, and the number on the bottom is the number of things chosen, or two. Using the word ‘choose’ is a language choice mathematicians make; it’s helpful to use the same words as other mathematicians for consistency. When we say ‘choose’ it also means the order of the items we select is irrelevant, for instance, in this problem, picking Edmund and Ferdinand is the same as picking Ferdinand and Edmund.”

Mr. Collins grabbed a large sheet of paper from his briefcase as I finished my drink, tipping my face to the ceiling to get the last few drops.

“Let’s move on. Have you ever heard of Pascal’s Triangle?” asked Mr. Collins.

“I don’t think so,” I replied, scooping some peppermint foam out of the mug with my finger and licking it off.

Mr. Collins slid the large sheet of paper across the table.

“You must begin,” said Mr. Collins, as though recounting the tale of some great battle, “with the number one at the top of the triangle.” He scratched his nose and added, “It’s probably best to write it in the centre of the paper, near the top.”

"Okay," I said, writing the number as instructed.

**1**

"Now put two ones below it, each a little to the side of the first one," instructed Mr. Collins.

"Like this?" I asked, showing Mr. Collins the sheet of paper.

1

**1 1**

"Exactly," said Mr. Collins, "now here's how you build the rest of the triangle: each number in a row is the sum of the two numbers above it. So, for example, the number right below both of those ones you just wrote will be a two, since one plus one is two." He added a two to the sheet of paper.

1

1 1

**2**

"I think I understand," I said. "And what about the numbers beside the two in the third row, what will they be?"

"They will be ones," said Mr. Collins, "in fact, all the very outer numbers will be ones, all along both sides of the triangle. This doesn't change, regardless of how many lines you write in Pascal's Triangle."

I wrote a bunch of ones. And writing those ones made sense since they were the sum of the only number above them, which was also one.

1

1 1

**1** 2 **1**

**1** **1**

**1** **1**

"Do you understand the pattern you can use to create the triangle?" asked Mr. Collins.

"Yes," I said. "Each number is the sum of the two numbers above it." During my sessions with Mr. Collins, I had worked with many patterns and this one, so far, seemed pretty simple.

"Perfect," said Mr. Collins. "Can you write in the fourth row now?"

The fourth row had to be one, three, three, one. The numbers in the middle had to both be threes since 1+2=3 and 2+1=3.

1

1 1

1 2 1

**1 3 3 1**

1 1

"Like this?" I asked. It really did look like a triangle now.

"Good work, Bethany," said Mr. Collins, "I think you've gotten it, just to make sure, what would the next row be?"

I looked at the last row I'd written, 1, 3, 3, 1. Of course, the first and last numbers of this new row were also one. I added 1+3=4 to get the second number in this row, 3+3=6 to get the third number, and 3+1=4 for the fourth number. The row was one, four, six, four, one.

1

1 1

1 2 1

1 3 3 1

**1 4 6 4 1**

I noticed that the fifth row was the same backwards and forwards. It was a palindrome, like the words RADAR and RACECAR. This made sense since each number was the sum of the two numbers above it, and so if the fourth row was symmetrical (and it was), then the fifth row would be as well and so on.

"Can you fill in the next three rows?" asked Mr. Collins.

It was pretty simple. I could just add the numbers from the rows before. This triangle could go on forever.

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

**1 5 10 10 5 1**

**1 6 15 20 15 6 1**

**1 7 21 35 35 21 7 1**

"Now," said Mr. Collins, "I hinted that there are some interesting patterns in Pascal's Triangle. I'm going to show you what I mean. Now, let's bring some of our ideas together. Remembering the notation of 'something choose something', can you tell me the number of ways we could pick two of your four friends?"

"*Four choose two*," I said, writing down $\binom{4}{2}$.

"That is correct," said Mr. Collins. "And do you remember what number four choose two is?"

"It's six, right? Since I found six ways to give two tickets to my four female friends. And same with my male friends."

"Yes. Now you'd agree with me that, from what you said, that four choose two equals six?"

I nodded.

"Can you find the first time six appears in Pascal's Triangle?"

I pointed to the six in the middle of the fifth row. "It's the third number in the fifth row."

1

1 1

1 2 1

1 3 3 1

1 4 **6** 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

"Yes, it is," said Mr. Collins. "I want you to think about whether that suggests any sort of pattern."

I flipped over the paper with Pascal's Triangle and wrote down what I had just observed, with words and then with numbers.

Fifth row, third number = four choose two

$5^{th}$ row, $3^{rd}$ number = $\binom{4}{2}$

One pattern was instantly clear to me. The fifth row, or the number five, was one more than the number of total things, four. The same was true for the number's placement in the row, third, or the number three, and the number of things chosen, two. I explained this to Mr. Collins.

"Very good," he replied, "but that's only a pattern that suggests itself, we need more evidence to see if it's true. See how at the top, the triangle starts with one? Well, one is equivalent to a lot of different things, but in this case, let's say it is $\binom{0}{0}$."

"Okay," I said. That $\binom{0}{0}$ equaled one made sense because if I had nothing and selected nothing from it, there was only one possible way to do this. I smiled.

"Let's move on to the second row," said Mr. Collins. "What should the numbers represent if your pattern is correct?"

"Well," I said. "Using my pattern for the second row, the total number of things to select should be one, since it's one less than the row number. So, the second row represents $\binom{1}{0}$ and $\binom{1}{1}$, right?"

"Very good," said Mr. Collins, "that would make sense according to your pattern, but do the actual numbers in the second row, that is, one and one, make sense for this?"

I thought about it. If I had one thing and selected nothing, I could only do this in one way. I figured this principle held true for any number choose zero. I moved on to the second number. If I picked one thing from one thing, I could do this in only one way.

"Yes," I said. "My pattern works."

"What about for the third row?" asked Mr. Collins.

If my pattern worked, and by this point I was pretty sure it did, I figured the third row should be equal to $\binom{2}{0}$, $\binom{2}{1}$, $\binom{2}{2}$. I already knew $\binom{2}{0}$ was one, since I'd figured out that anything choose zero should be one. That $\binom{2}{1}$ was two made sense, since if I had two things and picked one, I could do this in two ways. Finally, $\binom{2}{2}$ could only be done in one way, since I'd have to pick both items, so this was clearly equal to one.

"Yes, the pattern still works."

"Fantastic, Bethany. In fact, your pattern holds true for all of Pascal's Triangle."

"Okay," I said. "That's pretty neat."

"I think so too," said Mr. Collins, smiling. He glanced out at the snow, over at Joe who was preparing more drinks, and then back to me, "Bethany, it's almost Christmas."

"Yes, I know, I'm looking forward to it. Mom gets time off work and we are going over to Grandpa's for Christmas dinner. I think it's going to be fun."

"That sounds lovely. Say, do you know the song 'The Twelve Days of Christmas'?"

"Yeah," I said, raising an eyebrow. "The one about the true love and the partridge in the pear tree?" Of course, we all knew the song. In Cape Breton, Christmas carols were played from November onwards in the mall, and my school choir had their own special version of 'The Twelve Days of Christmas' complete with actions we all thought were ridiculous.

"So, the song says on the first day the true love sends one gift, 'a partridge in a pear tree'," said Mr. Collins. "And on the second day, it's 'two turtle doves, and a partridge in a pear tree' and so on until on the twelfth day. On the twelfth day, the true love sends twelve of something, like 'pipers piping' or 'lords a-leaping'."

"It's 'drummers drumming'," I interjected.

"That's impressive," said Mr. Collins, laughing. "Anyhow, here's my question: how many gifts does the true love send?"

"Can I just add them all up?" I asked. The question seemed pretty simple.

"Well, you could, but there's a more elegant way to do it. And besides, if you just add them all up, you could make a calculation error."

Mr. Collins gave me another index card. “Can you make a table for the first four days of Christmas?”

I drew a table.

| Day of Christmas | Sum of Gifts | Total |
|---|---|---|
| First | 1 | 1 |
| Second | 1+2 | 3 |
| Third | 1+2+3 | 6 |
| Fourth | 1+2+3+4 | 10 |

“How’s that?” I asked.

“Excellent,” said Mr. Collins, “Now let’s go back to Pascal’s Triangle. Where do you see the numbers that are the total numbers of gifts? Do you see anything interesting about where these numbers are?”

I looked at Pascal’s Triangle. The total numbers of gifts, one, three, six and ten, were in multiple places.

**1**<br>
**1** **1**<br>
**1** 2 **1**<br>
**1** **3** **3** **1**<br>
**1** 4 **6** 4 **1**<br>
**1** 5 **10** **10** 5 **1**<br>
**1** **6** 15 20 15 **6** **1**<br>
**1** 7 21 35 35 21 7 **1**

I noticed that there was an X-shape formed by two lines with the numbers one, three, six and ten. That seemed worth mentioning.

“Mr. Collins,” I said, “there are two lines with one, three, six and ten.” I pointed to the X-shape right in the centre of Pascal’s triangle.

“Excellent, Bethany. That is exactly what I wanted you to discover. We could do the problem with either of these lines, but let’s chose the one that starts on the right of the triangle.” He drew his finger down the line.

1

1 1

1 2 **1**

1 3 **3** 1

1 4 **6** 4 1

1 5 **10** 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

"Now, any guesses how many presents the true love receives on the fifth day of Christmas?"

My table didn't extend to the fifth day and besides Mr. Collins wanted me to guess, not calculate. I figured that the pattern must be the line and so the gifts for the fifth day should be the next number in the line. It was easy to see this number was 15.

"Is it 15?" I asked.

"Why don't you tell me?" said Mr. Collins, with a chuckle.

"Okay," I said, smiling. It had to be 1+2+3+4+5, which certainly added to 15.

"Yes, it is. So, this pattern continues in a line?" I asked, "All the way to the twelfth day?"

"That's right. Every day for the twelve days the true love sends the same number of gifts as the previous day *plus* a number of gifts equal to that day. For instance, on the fifth day, the true love sends the same number of gifts as on the fourth day *plus* five gifts. Remembering how you built Pascal's Triangle, does it make sense that the pattern would continue?"

Ten, the number of gifts on the fourth day, was in the diagonal line of gifts I had found. This ten added to the five beside it, making 15 the next number in the line of gifts. Then I could see that six would be added to 15 to make 21 the next number in the diagonal line that formed this pattern.

"Yeah," I said, "I think I understand. Let's see, to get the next number in the line of gifts, I always add an existing number from the line with the number beside it and to the left."

"And how does connect that to 'The Twelve Days of Christmas'?"

"Well, the number that you add to the existing one, that is to say the one that is to the left of it, that number, is always the same as the number of that day of Christmas."

"You mean like five for the fifth day?"

"Yeah, that's exactly what I mean," I said.

"Okay," said Mr. Collins, "and have you convinced yourself that the pattern always works?"

"I think so," I replied, "Since you always add the number of the day and the number of gifts from the previous day, the pattern I found where that diagonal line represents the number of gifts should always work."

"You're right, Bethany," said Mr. Collins. "And we're almost done. There's one last pattern I want to show you in Pascal's Triangle, called the 'Hockey Stick Identity'."

"Is that the real name?"

"It's what I call it," said Mr. Collins.

"Okay," I said, shrugging my shoulders and smiling.

"In the 'Hockey Stick Identity', the shape looks like a hockey stick and it involves some neat sums. Let me show you." Mr. Collins traced an L-shape on Pascal's Triangle.

1<br>
1 **1**<br>
1 **2** 1<br>
1 **3** 3 1<br>
1 4 **6** 4 1<br>
1 5 10 10 5 1<br>
1 6 15 20 15 6 1<br>
1 7 21 35 35 21 7 1

"Can you see how the numbers on the 'shaft' of the 'hockey stick' add to the number on the 'blade'?"

Mr. Collins looked out at the snow again. His eyes had a faraway look as though he were remembering hockey games of winters past. I imagined a much younger Mr. Collins skating on a frozen pond and yelling at his teammates, and I smiled.

I traced the hockey stick Mr. Collins had drawn. “I don’t know that much about hockey,” I said, laughing. “But I can see that 1+2+3=6. And the pattern looks pretty neat.”

“Yes, it is,” replied Mr. Collins. “And it always works, provided that you begin at the outside of Pascal’s Triangle. Can you find another ‘hockey stick’?”

“Sure,” I said. As long as I started at the outside of the triangle, I could make a line as long or short as I wanted and then find the line’s sum using the row below. I picked a random diagonal line of numbers: 1, 4, 10 and 20. To find their sum, I moved my finger to the row below and the number one over, 35.

1

1 1

1 2 1

1 3 3 **1**

1 4 6 **4** 1

1 5 10 **10** 5 1

1 6 15 **20** 15 6 1

1 7 21 35 **35** 21 7 1

“Did you make it work?” asked Mr. Collins with a twinkle in his eye.

“Yeah, in the case I picked 1+4+10+20=35, so the pattern seems to be true.”

“Excellent. Now how might this be useful to the ‘The Twelve Days of Christmas’ problem? Could we, say, find the total number of gifts, without actually adding them up?”

“I think so,” I said slowly. “I could continue the line of gifts to the twelfth day and then go down one row and over one to create a ‘hockey stick’. That would work because of the ‘Hockey Stick Identity’, right?”

“It certainly would work,” said Mr. Collins. “Good work, Bethany.”

He pulled another sheet of paper out of his briefcase.

“In the interest of time,” he continued, “I have printed out a much longer Pascal’s Triangle; it has more numbers than the one you created. I know you could extend it, but I want your effort spent observing the beauty of Pascal’s Triangle and its patterns and their applications, not doing simple calculations.”

I looked at the bigger Pascal's Triangle. "Thanks, Mr. Collins, some of these numbers are really huge." I smiled, "I can see the line of gifts."

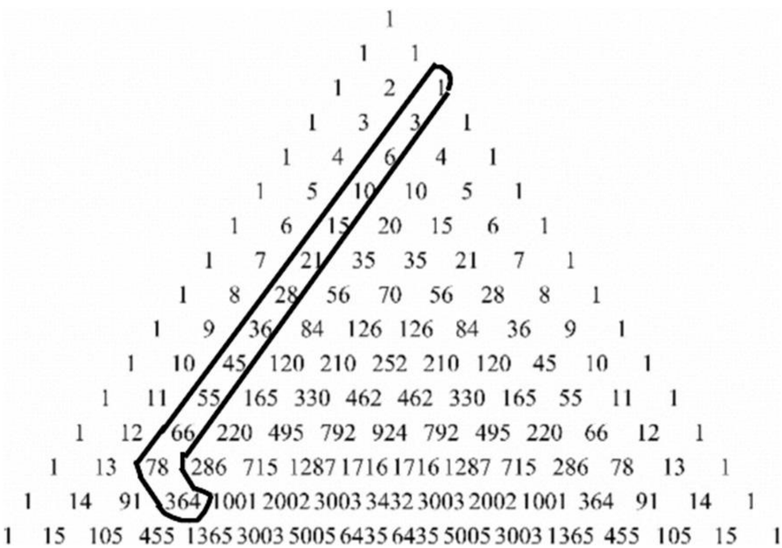
"Excellent," said Mr. Collins. "Show me."

"Well, it should continue to the twelfth day, right?"

"Yes, it should," said Mr. Collins.

I circled the line of gifts on the paper and went down to the next row and over one to make the 'blade' of the "hockey stick". I now knew the answer. I knew how many gifts the true love sent.

"It's 364!" I exclaimed. "That's a lot of gifts."




"Okay," said Mr. Collins, "Let's verify your answer using multiplication. The true love gives 'a partridge in a pear tree' on each of the twelve days. Would you agree this can be represented by the product 1×12?"

"Definitely," I said, clapping my hands together. "And then the 'two turtle doves' are 2×11, right?"

"Yes," said Mr. Collins, "I think you've got it."

"Well, all of the gifts can be thought of like this, from 1×12, like you said for the 'partridge in a pear tree' all the way to 12×1 for the 'drummers drumming' because the true love only sends the drummers on the twelfth day of Christmas."

“Fantastic, Bethany,” said Mr. Collins, reaching into his leather briefcase again and pulling out a calculator, which he handed it to me.

I entered (1×12) + (2×11) + (3×10) + (4×9) + (5×8) + (6×7) + (7×6) + (8×5) + (9×4) + (10×1) + (11×2) + (12×1) into the calculator.

The answer *was* 364!

“Congratulations,” said Mr. Collins. “Well done. It is indeed 364. You’ve had many wonderful insights today. Think about how far we’ve come just in this session. We remembered the counting rectangle problem, and gave it a new context with friends and plane tickets, we created Pascal’s Triangle ourselves and using its patterns found how many gifts the true love sent in ‘The Twelve Days of Christmas’. Isn’t that amazing?”

I looked outside at the snow. I felt the warmth of Le Bistro, and the kindness of my mentor, Mr. Collins.

I decided that, yes, it was amazing – all of it.